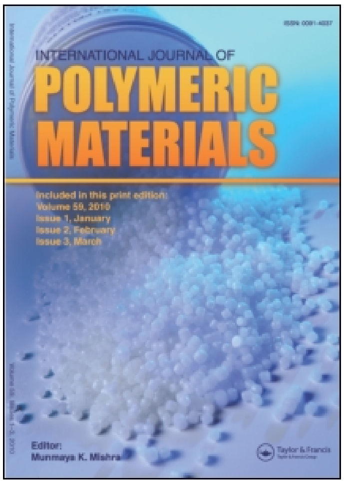

International Journal of Polymeric Materials
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713647664

Contact Methods for Creep and Elasticity Investigations in Polymer Mechanics

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To cite this Article Wilczyński, Andrew P.(1979) 'Contact Methods for Creep and Elasticity Investigations in Polymer Mechanics', International Journal of Polymeric Materials, 7: 3, 185 — 190

To link to this Article: DOI: 10.1080/00914037908077925

URL: http://dx.doi.org/10.1080/00914037908077925

Intern. J. Polymeric Mater., 1979, Vol. 7, pp. 185–190
0091–4037/79/0703–0185 $04.50/0

Printed in Reading, England

Contact Methods for Creep and Elasticity Investigations in Polymer Mechanics

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(*Received February 20, 1978*)

A new method for measuring values of elastic and creep moduli of plastics is proposed. The method is based on an observed phenomenon found while investigating behaviour of compressed hollow discs. According to this observation a ratio of internal and external diameters has been found, ensuring a displacement of a diameter of the disc depending on one material compliance only. Outlines of a theoretical solution and some experimental results are given.

INTRODUCTION

When polymers are to be used in technical applications their mechanical behaviour can be described with sufficient accuracy by the linear viscoelastic theory. However, the majority of existing equipment is unsuitable for the determination of the material constants required for the solution of the equations. This problem is further complicated when performance over a range of temperature is required.

A simple method of measurement is proposed in this paper which involves the design and manufacture of small testers, which can be totally enclosed in a climatic cabinet. This method is based on an observed theoretically interesting phenomenon that the increase of a diameter perpendicular to the direction of compression of a hollow disc can, for certain ratios of diameters, depend only on the shear modulus G of the material and does not depend on other elastic constants.

PRINCIPLES OF MEASUREMENT

A circular hollow disc in plane stress of arbitrary radii r and R is considered to be compressed between two rigid plates as depicted in Figure 1, while measurements are performed in perpendicular direction to loading to avoid the influence of stress concentrations and yielding zones. It can be shown[1] that the increase in diameter perpendicular to the direction of loading is a function of $x = r/R$ and the material constants E and G for an isotropic material. From theoretical computed results an interesting phenomenon was observed. For a specific value of $x = x_s$ the increase in diameter is a function of the constant G only. Therefore by testing a sample having a ratio of radii $x = x_s$

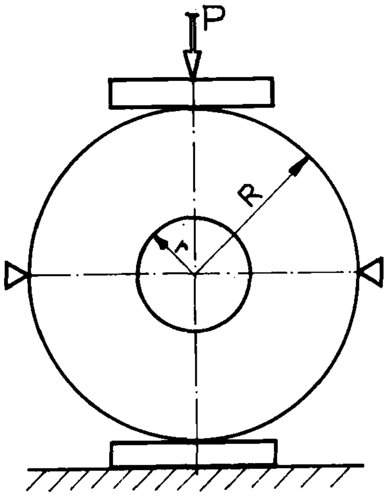

FIGURE 1 Principles of measurement.

the value of G can be calculated. From a second sample, for which $x \neq x_s$ the value of E can be determined by using the known value of G. In particular and to simplify the experimental method $x = 0$ is found to be most convenient for the second sample. The two specimens are otherwise identical. When the values of E and G are recorded as functions of time, they represent the creep moduli, provided that the time of measurement is sufficiently large when compared with time of initial loading. This statement follows from a simple consideration, presented in Figure 2. The "real" creep curve 3, depending on the actual path of loading lies between two "theoretical" creep curves 1 and 2, shifted horizontally by the time of loading. As the slope of a creep curve tends to zero with increase of time all three curves coincide with unsignificant error for large values of time.

THEORETICAL

It has been shown[1] that an exact solution of the contact problem presented in Figure 1 can be obtained. The outlines of this treatment are as follows: a

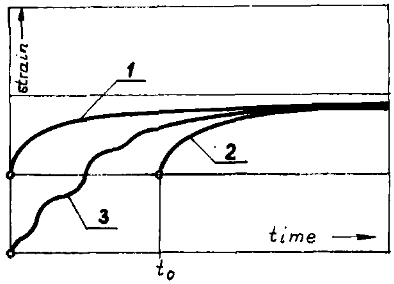

FIGURE 2 Initial loading error in creep.

solution for a disc loaded radially with arbitrary self-balanced radial load in a form of Fourier series can be found.[2] A Fourier representation of a Dirac's $\delta(\pi n)$ function was considered. If this representation is multiplied by the load P, the solution corresponds to a solid disc compressed as in Figure 1. Such treatment could be extended accordingly to a hollow disc. This case has been solved in analytical form but final results had to be computerized.[1]

For a solid disc ($x = 0$) the increase of diameter can be written in the form

$$u_1 = 0.72676 \frac{P}{b}\left(\frac{1}{E} - 0.68798 \frac{1}{G}\right) \tag{1}$$

where P is the acting force and b is the thickness of the plate. For a hollow plate the relation can be written in the form

$$u_2 = 0.72676 \frac{P}{b}\left(\frac{\beta}{E} - 0.68798 \frac{1}{G}\right) \tag{2}$$

where β is a function of x only. This function given in[1] can be approximated for the values of $0.24 \leqslant x \leqslant 0.28$ with an insignificant error as

$$\beta(x) = 2.517 - 9.832\,x \tag{3}$$

For a value of $x_s = 0.256$, $\beta = 0$ and finally for this case

$$u_2 = -0.5 \frac{P}{bG} \tag{4}$$

Using relations (1) and (4) it is easily found that

$$G(t) = -0.5\frac{P}{bu_2}$$
$$E(t) = 0.72676 \frac{P}{b(u_1 - u_2)} \tag{5}$$

It can be proved numerically that relations of this type are correct for the displacements perpendicular to the direction of loading and with unsignificant error (0.01 % and less) are independent of the radius of the disc and eventual yielding zone of contact.

EXPERIMENTAL

The specimen was selected as a disc of external diameter 50.8 mm with thickness b of 10 mm. It was found that a thickness of 0.2 ÷ 0.3 of the diameter is best suitable for experiments. This ratio prevents the disc from buckling. Figure 3 gives a general view of a small tester which was prepared for creep

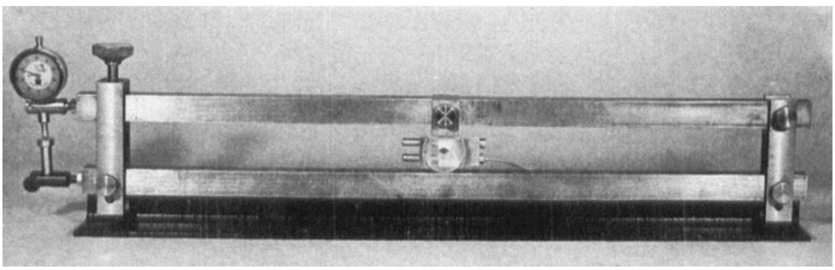

FIGURE 3 A small tester to be enclosed in a climatic cabinet together with a specimen and measuring device.

tests at elevated temperatures. The measuring device consists of an inductive transducer held by friction grips to the outer surface of the disc with a compressive force of about 0.5 N.

It is sometimes convenient to employ the same sample for both specimens, first as a solid and then with a drilled hole of the appropriate size. This has proved to give satisfactory results but it is suggested that the direction of loading be changed by 90°.

The experiments were performed with polyvinylchloride and polymethylmethacrylate (PMMA). The specimen diameter change was measured after 30 seconds from the moment of loading at a temperature of 22°C. Loading was accomplished using a commercial Rockwell hardness tester while the change in diameter was measured with an inductive extensometer shown in Figure 4. Data from the experiment performed with PMMA are given in Table I.

A tensile test, using strain gauges for measurement of longitudinal and transversial strains, gave values of $\hat{E} = 3127$ MN/m^2 and $G = 1091$ MN/m^2 while results calculated from Table I give $E(t) = 3162$ MN/m^2 and $G(t) = 1159$ MN/m^2. This leads to errors of 1.12% and 5.80% respectively. The

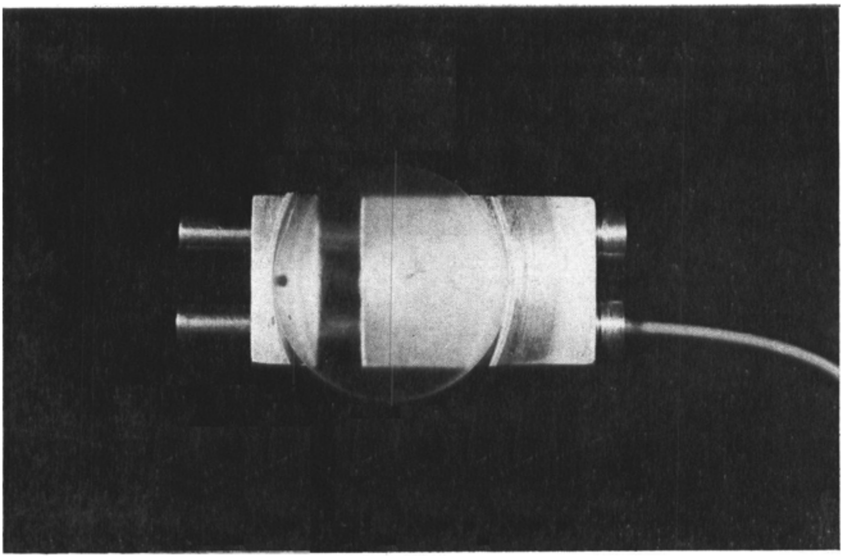

FIGURE 4 An inductive measuring device with a specimen held by friction.

TABLE I
Increase in diameter at various loads for PMMA

	Solid disc u_1 [μm]			Hollow disc u_2 [μm]		
	Load [N]			Load [N]		
Test N_0	250	500	900	250	500	900
1	5	10	19	11	22	39
2	6	10	18	10	22	41
3	6	11	18	11	20	39

material was selected as nearly purely elastic at room temperature, not being sensitive to speed of loading and exhibiting low rate of creep.

When testing other materials (e.g. PVC) a pronounced creep behaviour was observed.

CONCLUSIONS

The theoretical results were confirmed experimentally with good agreement. The method presented, designed mainly for creep tests, proved itself convenient for elastic constants measurements. It seems at present that the only disadvantage of the proposed method lies in its inapplicability to nonlinear behaviour measurements so its validity holds for all materials which can be described as linear, either elastic or viscoelastic in the region of small strains at constant temperatures.

References

1. H. Olesiak, A. P. Wilczyński, *Mech. Teoret. i Stos.* **3**, 13, 353–361 , (1975) (in Polish)
2. K. Girkmann, *Flachentragwerke*, Springer (1956).